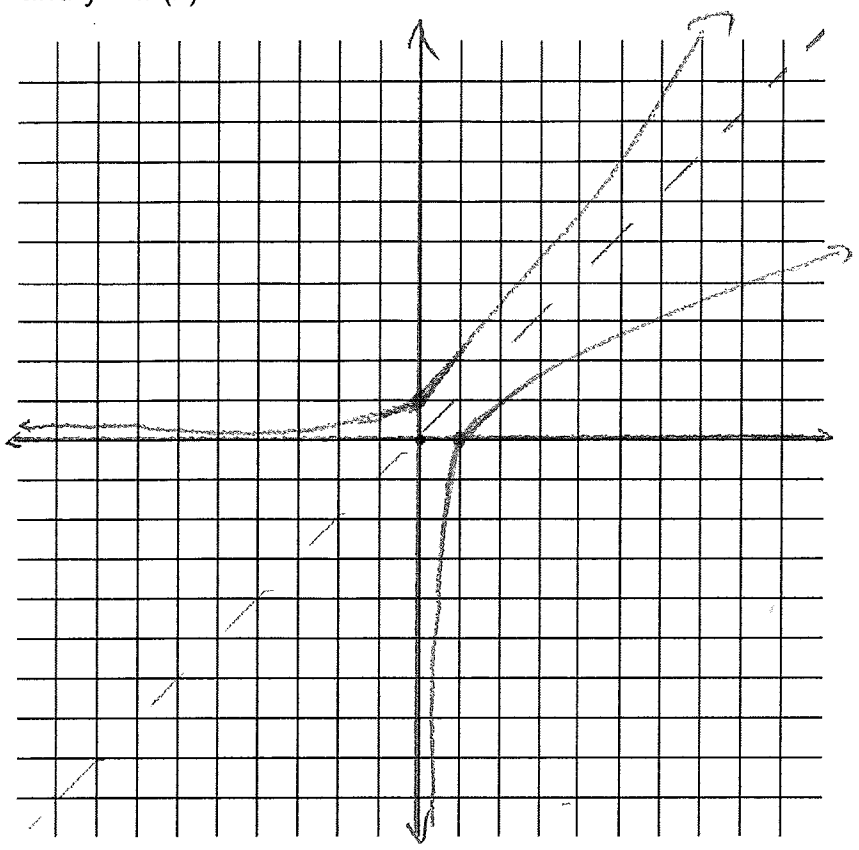
Mar Qureshi

Key Features of the Natural Logarithm

Graph $y = e^x$ and $y = \ln(x)$ on the same set of axes:



Summarize the key features of both graphs on the table below:

	$y = e^x$	$y = \ln(x)$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R} \mid x > 0$
Range	$y \in \mathbb{R} \mid y > 0$	$y \in \mathbb{R}$
y-intercept	$y = 1$	none
Increasing/decreasing intervals	always increasing	
x-intercept	none	$x = 1$
Asymptotes	horizontal at $y = 0$	vertical at $x = 0$
Min/max points	none	none
Inflection points	none	none
concavity	up	down

A Practise

Use this information to answer questions 1 to 3.

Let $f(x) = -e^x$ *and* $g(x) = -\ln x$.

1. a) Use technology to graph $f(x)$.

   b) Identify the following key features of the graph.

   i) domain

   ii) range

   iii) any x- or y-intercepts

   iv) the equations of any asymptotes

   v) intervals for which the function is increasing or decreasing

   vi) any minimum or maximum points

   vii) any inflection points

2. Repeat question 1 for $g(x)$.

3. Are $f(x)$ and $g(x)$ inverse functions? Justify your answer with mathematical reasoning.

4. Estimate the value of each exponential function, without using a calculator.

   a) e^4 b) e^5 c) e^2 d) e^{-2}

5. Evaluate each expression in question 4, correct to three decimal places, using a calculator.

6. Evaluate, if possible, correct to three decimal places, using a calculator.

   a) $\ln 7$ b) $\ln 200$ c) $\ln \frac{1}{4}$ d) $\ln(-4)$

7. What is the value of $\ln 0$? Why is this reasonable?

B Connect and Apply

8. Simplify.

   a) $\ln(e^{2x})$

   b) $\ln(e^x) + \ln(e^x)$

   c) $e^{\ln(x+1)}$

   d) $(e^{\ln(3x)})(\ln(e^{2x}))$

9. Solve for x, correct to three decimal places.

   a) $e^x = 5$

   b) $1000 = 20e^{\frac{x}{4}}$

   c) $\ln(e^x) = 0.442$

   d) $7.316 = e^{\ln(2x)}$

10. a) Solve $3^x = 15$ by taking the natural logarithm of both sides.

    b) Solve $3^x = 15$ by taking the common logarithm (base 10) of both sides.

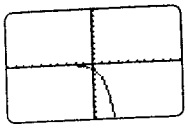
    c) What do you conclude?

11. Chapter Problem Sheona's supervisor has given her some capacitors to analyse. When one of the charged capacitors is connected to a resistor to form an RC (resistor-capacitor) circuit, the capacitor discharges according to the equation $V(t) = V_{max}e^{-\frac{t}{4}}$, where V is the voltage, in volts; t is time, in seconds; and V_{max} is the initial voltage, in volts. Determine how long it will take for a capacitor in this type of circuit to discharge to

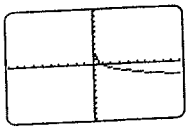
    a) half of its initial charge

    b) 10% of its initial charge

1. a) $f(x) = -e^x$ b) i) $x \in \mathbb{R}$ ii) $\{y | y \in \mathbb{R}, y < 0\}$ iii) x-intercepts: none; y-intercept: -1 iv) horizontal asymptote: $y = 0$ v) decreasing: $x \in \mathbb{R}$ vi) no maximum or minimum points vii) no points of inflection



2. a) $g(x) = -\ln x$ b) i) $\{x | x \in \mathbb{R}, x > 0\}$ ii) $y \in \mathbb{R}$ iii) x-intercept: 1; y-intercepts: none iv) vertical asymptote: $x = 0$ v) decreasing: $\{x | x \in \mathbb{R}, x > 0\}$ vi) no maximum or minimum points vii) no points of inflection

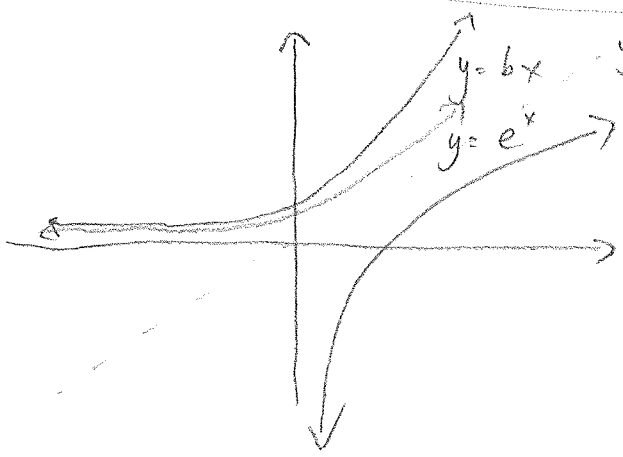


3. Answers may vary. No. $f(x)$ and $g(x)$ are not inverse functions. They are not reflections of each other in the line $y = x$. 4. Answers may vary. a) 55 b) 150 c) 7.5 d) 0.1 5. a) 54.598 b) 148.413 c) 7.389 d) 0.135 6. a) 1.946 b) 5.298 c) -1.386 d) undefined 7. Undefined. Answers may vary. The domain of the function $y = \ln x$ is $\{x | x \in \mathbb{R}, x > 0\}$. Therefore, the value of $\ln 0$ is undefined. 8. a) $2x$ b) $2x$ c) $x + 1$ d) $6x^2$ 9. a) 1.609 b) 15.648 c) 0.442 d) 3.658 10. a) $x = 2.465$ b) $x = 2.465$ c) Answers may vary. The value of x can be found by taking natural logarithms of both sides of the equation or by taking common logarithms of both sides of the equation. 11. a) 2.8 s b) 9.2 s 12. a) $k = 7$ b) $T(10) = 48°C$ c) $T(15) = 23°C$. Answers may vary.

Moe ~
Quarter

The Natural Logarithm

bored!



$x = e^y$ rewrite as a log

$\log_e x = y$

$\ln x = y$ → natural logarithm

Ex) the population of Indonesia as a function of time (in yrs after 2000) is given by $P(t) = 221 e^{0.01499t}$ (in millions)

a) Determine the initial population

$P(0) = 221 e^{0.01499(0)}$

$= 221$ million

b) Determine the population in 2012

$P(12) = 221 e^{0.01499(12)}$

$= 264$ million

c) How long will it take the population to double?

$442 = 221 e^{0.01499t}$

$2 = e^{0.01499t}$

$\ln 2 = \ln e^{0.01499t}$

$t = 46.6$

d) Rewrite the population function with base 2

$P(t) = 221 e^{0.01499t}$

$P = P_0(2)^{t/D}$ ← doubling

$P(t) = 221\left(2^{t/46.6}\right)$

Half life $M = M_0\left(\frac{1}{2}\right)^{t/H}$