

Grade 11 Skills – Solving Equations

Exercises

Solve each of the following equations. The exercises are grouped according to the type of equation being solved.

1–8 Simple Linear equations

1. $3x - 5 = 11$

2. $4x + 3 = 2x - 7$

3. $-3x + 8 = 5x - 2$

4. $2x - \frac{1}{5} = \frac{3}{5}$

5. $\frac{3x+7}{9} = 5$

6. $\frac{8-3x}{7} = \frac{x+1}{4}$

7. $\frac{2+5x}{3} + \frac{x-1}{7} = -2$

8. $\frac{x+2}{5} - \frac{x-3}{6} + \frac{x+1}{15} = \frac{x-4}{2}$

9–13 Linear Equations with Literal Constants

9. $ax = b \quad (a, b \in \mathbb{R}, a \neq 0)$

10. $ax + 3 = b \quad (a, b \in \mathbb{R}, a \neq 0)$

11. $ax - 5 = b \quad (a, b \in \mathbb{R}, a \neq 0)$

12. $ax + b = c \quad (a, b, c \in \mathbb{R}, a \neq 0)$

13. $\frac{ax+b}{c} = \frac{d}{e} \quad (a, b, c, d \in \mathbb{R}, a, c, e \neq 0)$

14–29 Equations Involving Rational Expressions (variable in the denominator)

14. $\frac{3}{x} = 7$

15. $-\frac{4}{x} + 5 = -1$

16. $\frac{5}{x+2} = \frac{1}{3}$

17. $\frac{2}{x} = \frac{7}{x+1}$

18. $\frac{2x+1}{3x-5} = 6$

19. $\frac{5-7x}{7-5x} = \frac{7}{5}$

20. $\frac{x-3}{x+2} + 3 = 2$

21. $\frac{2x-4}{3-2x} = \frac{7}{3}$

22. $\frac{-13-3x}{x+5} = \frac{2}{x+5} + 3$

23. $\frac{3}{2x-1} + \frac{5}{2x-1} = \frac{12}{4x-2}$

24. $\frac{-2}{x+9} + \frac{7}{-x-9} = \frac{12}{19}$

25. $\frac{4x+4}{x^2+3x+2} - \frac{2}{x+2} = \frac{3}{8}$

26. $\frac{x^2-9}{6x-18} + \frac{3x^2+5x-2}{5x+10} = \frac{7}{30}$

27. $\frac{x^2+x}{x+1} + \frac{x^2+2x+1}{(x+1)^2} = 0$

28. $\frac{5x^2-4x-28}{(x+3)(x+2)(5x-14)} + \frac{x-7}{x^2-4x-21} = -1$

29. $\frac{(x^2-7x+10)(x^2-2x-15)}{(x^2+x-6)(x^2-10x+25)} + 1 = \frac{6x-60}{3x-30}$

30–39 Equations Involving Rational Expressions and Literal Constants

30. $\frac{a}{x} = 10 \quad (a \in \mathbb{R}, a \neq 0)$

31. $-\frac{a}{x} + b = 5 \quad (a, b \in \mathbb{R}, a \neq 0)$

32. $\frac{a}{x+b} = \frac{c}{d} \quad (a, b, c, d \in \mathbb{R}, a, d \neq 0)$

33. $\frac{a}{x} = \frac{b}{x+c} \quad (a, b, c \in \mathbb{R}, a, b \neq 0)$

34. $\frac{a}{x} + \frac{b}{2x} = \frac{c}{x+d} \quad (a, b, c, d \in \mathbb{R}, a, b, c \neq 0)$

35. $\frac{ax+b}{cx+d} = e \quad (a, b, c, d, e \in \mathbb{R}, a, b \neq 0)$

36. $\frac{ax+b}{cx+d} = e + f \quad (a, b, \dots, f \in \mathbb{R}, a, c \neq 0)$

37. $\frac{ax+b}{cx+d} = \frac{e}{f} \quad (a, b, \dots, f \in \mathbb{R}, a, c, f \neq 0)$

38. $\frac{ax+b}{cx+d} = \frac{e}{f} + \frac{g}{h} \quad (a, b, \dots, h \in \mathbb{R}, a, c, f, h \neq 0)$

39. $\frac{ax+b}{cx+d} = \frac{x}{cx+d} \quad (a, b, c, d \in \mathbb{R}, a, c \neq 0)$

40–62 Quadratic Equations

Try to select the best method to solve each quadratic.

Remember not all quadratic equations have real solutions.

40. $x^2 = 25$

41. $x^2 - 36 = 0$

42. $x^2 - 12 = 0$

43. $x^2 = -16$

44. $x^2 + 49 = 0$

45. $x^2 - 3x - 10 = 0$

46. $x^2 + 6x + 5 = 0$

47. $x^2 + 12x = 28$

48. $x^2 - 10x + 25 = 0$

49. $x^2 + 9x + \frac{81}{4} = 0$

50. $3x^2 + 12x - 36 = 0$

51. $-5x^2 = 15x + \frac{45}{4}$

52. $2x^2 + 5x - 8 = 0$

53. $3x^2 - 2x + 15 = 0$

54. $(x + 1)^2 = 36$

55. $3(x - 2)^2 = 12$

56. $5(2x - 1)^2 = 30$

57. $[2(x - 3)]^2 = 400$

58. $\left[\frac{2}{3}(3x - 1)\right]^2 = 400$

59. $\left[-\frac{4}{5}(1 - 5x)\right]^2 = 3$

60. $\left[-\frac{1}{3}\left(\frac{3}{5}x + 4\right)\right]^2 - 9 = 0$

61. $\left[-\frac{9}{7}\left(\frac{2}{3}x - \frac{1}{6}\right)\right]^2 - \frac{25}{49} = 0$

62. $\pi(x - 1)^2 = 9\pi$

63–70 Quadratic Equations with Literal Constants

63. $a(3x + 2)^2 = k^2a^3 \quad (a, k \in \mathbb{R}, a \neq 0)$

64. $x^2 + 2kx + k^2 = 0 \quad (k \in \mathbb{R})$

65. $a^2x^2 + 2akx + k^2 = 0 \quad (a, k \in \mathbb{R}, a \neq 0)$

66. $x^2 + (k - 3)x - 3k = 0 \quad (k \in \mathbb{R}, k \neq 0)$

67. $kx^2 - 3kx + 1 = 0 \quad (k \in \mathbb{R}, k \neq 0)$

68. $2kx^2 - (k + 1)x + (3k - 4) = 0 \quad (k \in \mathbb{R}, k \neq 0)$

69. $x^2 - kx + 3x + 7 - k = 0 \quad (k \in \mathbb{R})$

70. $-kx^2 + x - 5 + 2kx - 7 = 3 + kx \quad (k \in \mathbb{R}, k \neq 0)$

70–77 Equations requiring a combination of skills.

71. $\left(\frac{5}{x - 1}\right)^2 = 7$

72. $\left(\frac{x - 8}{4}\right)^2 = \left(\frac{2x - 1}{3}\right)^2$

73. $\left(\frac{1}{3x + 1}\right)^2 = \left(\frac{3}{x - 4}\right)^2$

74. $\frac{3x + 2}{x - 1} = \frac{x + 7}{x - 3}$

75. $x + \frac{x + 2}{x - 1} = \frac{7x + 3}{x - 1}$

76. $\frac{x}{2x - 3} - \frac{x + 1}{x + 2} = -\frac{25}{99}$

77. $\frac{x - 3}{x + 1} + \frac{x + 2}{x - 7} = -\frac{19}{6}$

78–88 Just Plain Silly.

78. $x^4 - 9x^2 + 18 = 0$

79. $x^4 - 2x^2y^2 + y^4 = 0 \quad (y \in \mathbb{R})$

80. $x^8 - 81 = 0$

81. $x^{10} - 121x^8 = 0$

82. $x^3 + 6x^2 - 40x = 0$

83. $\sqrt{x - 3} = 4$

84. $\sqrt{x + 2} = 7$

85. $5 - \sqrt{x} = \sqrt{x + 5}$

86. $9 + \sqrt{x - 1} = 20 - \sqrt{3x - 2}$

87. $\frac{(x - 1)^4}{x^2 + 12x + 36} = x^2 - 10x + 25$

88. $\frac{(x + 2)^6}{9x^2 + 36x + 36} = 25x^2 - 30x + 9$

Answers

1. $x = \frac{16}{3}$
2. $x = -5$
3. $x = \frac{5}{4}$
4. $x = \frac{2}{5}$
5. $x = \frac{38}{3}$
6. $x = \frac{25}{19}$
7. $x = -\frac{53}{38}$
8. $x = \frac{89}{12}$
9. $x = \frac{b}{a}$
10. $x = \frac{b-3}{a}$
11. $x = \frac{b+5}{a}$
12. $x = \frac{c-b}{a}$
13. $x = \frac{cd-eb}{ea}$
14. $x = \frac{3}{7}$
15. $x = \frac{2}{3}$
16. $x = 13$
17. $x = \frac{2}{5}$
18. $x = \frac{31}{16}$
19. no solutions exist
20. $x = \frac{1}{2}$
21. $x = \frac{33}{20}$
22. no solutions exist
23. no solutions exist
24. $x = -\frac{93}{4}$
25. $x = \frac{10}{3}$
26. $x = -\frac{2}{23}$
27. $x = -1$
28. $x = -5$
29. $x \in \mathbb{R}$
30. $x = \frac{a}{10}$
31. $x = \frac{a}{b-5}$
32. $x = \frac{ad-bc}{c}$
33. $x = -\frac{ac}{a-b}$
34. $x = \frac{-2ad-bd}{2a+b-2c}$
35. $x = \frac{b-ed}{ec-a}$
36. $x = \frac{b-df-ed}{-a+cf+ec}$
37. $x = \frac{bf-ed}{ec-af}$
38. $x = \frac{bfh-dfg-edh}{-afh+cfg+ech}$
39. $x = \frac{-b}{a-1}$
40. $x = \pm 5$
41. $x = \pm 6$
42. $x = \pm 2\sqrt{3}$
43. no real solutions
44. no real solutions
45. $x = 5$ or $x = -2$
46. $x = -5$ or $x = -1$
47. $x = -14$ or $x = 2$
48. $x = 5$
49. $x = -\frac{9}{2}$
50. $x = -6$ or $x = 2$
51. $x = -\frac{3}{2}$
52. $x = \frac{-5 \pm \sqrt{89}}{4}$
53. no real solutions
54. $x = -7$ or $x = 5$
55. $x = 0$ or $x = 4$
56. $x = \frac{5 \pm \sqrt{6}}{2}$
57. $x = -7$ or $x = 13$
58. $x = -\frac{29}{3}$ or $x = \frac{31}{3}$
59. $x = \frac{4 \pm 5\sqrt{3}}{20}$
60. $x = -\frac{65}{3}$ or $x = \frac{25}{3}$
61. $x = -\frac{7}{12}$ or $x = \frac{13}{12}$
62. $x = -2$ or $x = 4$
63. $x = \frac{-2 \pm ak}{3}$
64. $x = -k$
65. $x = -\frac{k}{a}$
66. $x = 3$ or $x = -k$
67. $x = \frac{3k \pm \sqrt{9k^2 - 4k}}{2k}$
68. $x = \frac{k+1 \pm \sqrt{-23k^2 + 34k + 1}}{4k}$
69. $x = \frac{k-3 \pm \sqrt{k^2 - 2k - 19}}{2}$
70. $x = \frac{k+1 \pm \sqrt{k^2 - 58k + 1}}{2k}$
71. $x = \frac{7 \pm 5\sqrt{7}}{7}$
72. $x = -4$ or $x = \frac{28}{11}$
73. $x = -\frac{7}{8}$ or $x = \frac{1}{10}$
74. $x = \frac{13 \pm \sqrt{161}}{4}$
75. $x = \frac{7 \pm \sqrt{53}}{2}$
76. $x = -\frac{3}{7}$ or $x = 7$
77. $x = \frac{1}{31}$ or $x = 5$
78. $x = \pm\sqrt{3}$ or $x = \pm\sqrt{6}$
79. $x = \pm y$
80. $x = \pm\sqrt{3}$
81. $x = \pm 11$ or $x = 0$
82. $x = -10$ or $x = 0$ or $x = 4$
83. $x = 19$
84. $x = 47$

Answers ... cont'd

85. $x = 4$

86. $x = 17$

87. $x = \frac{31}{3}$ or $x = \frac{1 \pm \sqrt{233}}{4}$

88. $x = \frac{11 \pm \sqrt{69}}{2}$ or $x = \frac{-19 \pm \sqrt{381}}{2}$