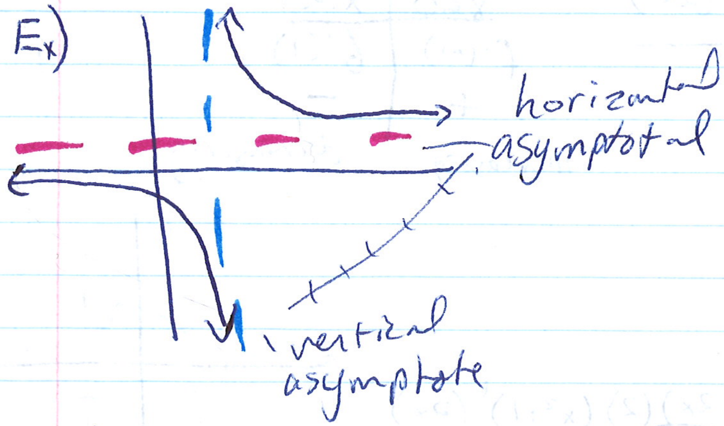
Moe Qureshi

Rational Functions

$R(x) = \frac{P(x)}{Q(x)}$, $Q(x)$

Ex)



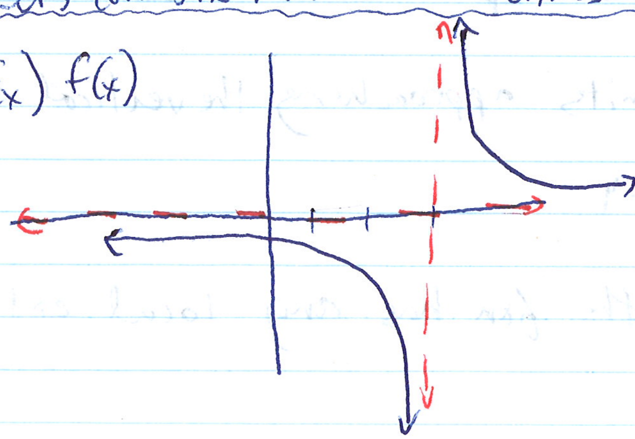
Vertical Asymptotes happen where the function is undefined (because the denominator would be zero)

Horizontal Asymptotes happen when:

1) degree of denom > degree of numerator $\boxed{e,\ y=0}$
2) degree of numerator > degree of denominator
   divide denom into numerator + throw away remainder
3) degrees are equal
   divide each leading coefficient

Let's combine Rational Functions with Limits

Ex) f(x)

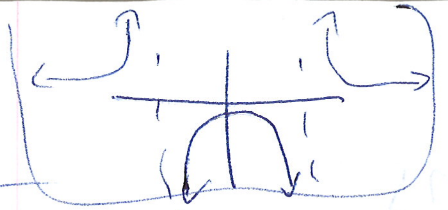


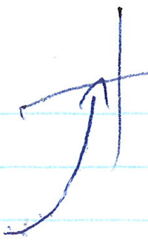
$f(x) = \frac{1}{x-3}$ $\lim_{x \to 3^-} \left(\frac{1}{x-3}\right) = -\infty$

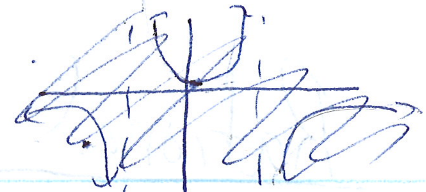
$\lim_{x \to 3^+} \left(\frac{1}{x-3}\right) = +\infty$

$\lim_{x \to \infty} \left(\frac{1}{x-3}\right) = 0$

$\lim_{x \to -\infty} \left(\frac{1}{x-3}\right) = 0$



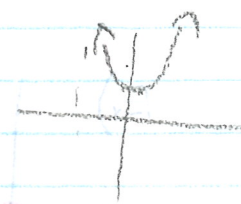




Ex) For $f(x) = \frac{1}{x^2+1}$, find the increasing and decreasing intervals and any points of inflection

$$f'(x) = \frac{0(x^2+1) - (1)(2x)}{(x^2+1)^2}$$

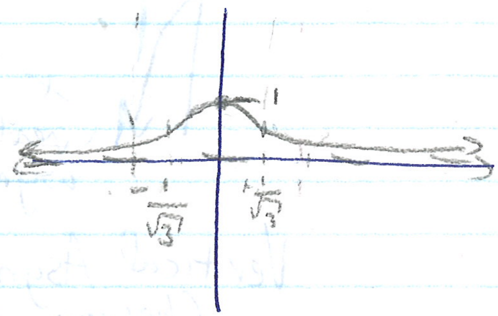
$x<0$	$x>0$
$f'(-1)$	$f'(1)$
$+$	$-$
increasing	decreasing



$$0 = \frac{-2x}{(x^2+1)^2}$$

$$\cancel{0} = \frac{-2x}{(x^2+1)^2}$$

$$x = 0$$



$$f''(x) = \frac{-2(x^2+1)^2 \cancel{+} (-2x)(2)(x^2+1)^1(2x)}{(x^2+1)^4}$$

$$f''(x) = \frac{-2(x^2+1)^{\cancel{2}1} + 8x^2\cancel{(x^2+1)}}{(x^2+1)^{\cancel{4}3}}$$

$$0 = \frac{6x^2-2}{(x^2+1)^3}$$

~~$x<-\sqrt{\frac{1}{3}}$~~	~~$x>-\sqrt{\frac{1}{3}}$~~	~~$x<+\sqrt{\frac{1}{3}}$~~	~~$x>+\sqrt{\frac{1}{3}}$~~

$$0 = 6x^2 - 2$$

$$\frac{2}{6} = x^2$$

$$x = \pm\sqrt{\frac{1}{3}}$$

$x<-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$	$x>\frac{1}{\sqrt{3}}$
$+$	$-$	$+$
concave up	concave down	concave up

HW ① Find the one sided limits approaching the vertical asymptotes of $f(x) = \frac{x-5}{x^2+2x-4}$

② Determine whether the fxn has any local extrema $f(x)$

$$f(x) = \frac{x}{x-4}$$